

NRO NSA in UK

By Anthony Shell 20th April 2024

A key role of the US military intelligence base at Menwith Hill (Yorkshire, UK) is assessed to be that of obtaining targetting data for use in the elimination of those within Eurasia who oppose US global hegemony.

ANALYSIS

[1] US MENWITH HILL, Yorkshire (UK) is an integral part of the US MISSION 8300 Program, under the command and operational control of: (a) NRO (adversarial individuals/groups acquisition, tracking, and targeting); and (b) NSA (content decryption and analysis of adversories' comms).

[2] US MISSION 8300 objectives are primarily: (1) of the targeted elimination of uncoperative/disobediant foreign political leaders and military commanders; and (2) of the destruction of their defence assets, and of the economic and civil infrastructure of those adversarial countries.

[3] MISSION 8300 objectives fullfilment requires use of advanced ORION satellite platforms, in 'figure-of-8' (inclined + eccentric) GEOSYNCH orbital paths, covering regions of special interest (i.e. EUCOM and CENTCOM) selected to be within the context of contemporaneous events.

[4] Targets - locatable by psuedo-range multilateration surveillance system implementation (i.e. does NOT require transmit times to be within intercepted RF signals) - include: cell-net phone/laptop comms; diplomatic/military comms; radars; defence jammers (i.e. GPS, Link 16); etc.

[5] Note - the switching on or off, of RF emitters, expedites the rapid determination of RF emitter location. MISSION 8300 combined ops may therefore include (for example) the use of deployed military assets such as to provoke adversaries into switching air defence radars on or off.

[6] This pseudo-range multilateration system is very substantially the same as that used for GPS (albeit with a somewhat different system architecture). The accuracy of targetting will therefore be that of GPS - i.e. (it is estimated) to be 1 meter CEP (2-D) and 3 meters SEP (3-D) (assuming number of satellites used is 4 or more).

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Analysis / verification of: 'Pseudo-range multilateration' wikipedia.com [1]

The TOT concept is illustrated in Figure 2 for the surveillance function and a planar scenario ($d = 2$). Aircraft A, at coordinates (x_A, y_A) , broadcasts a pulse sequence at time t_A . The broadcast is received at stations S_1 , S_2 and S_3 at times t_1 , t_2 and t_3 respectively. Based on the three measured TOAs, the processing algorithm computes an estimate of the TOT t_A , from which the range between the aircraft and the stations can be calculated. The aircraft coordinates (x_A, y_A) are then found.

When the algorithm computes the correct TOT, the three computed ranges have a common point of intersection which is the aircraft location (the solid-line circles in Figure 2). If the computed TOT is after the actual TOT, the computed ranges do not have a common point of intersection (dashed-line circles in Figure 2). It is clear that an iterative TOT algorithm can be found. In fact, GPS was developed using iterative TOT algorithms. Closed-form TOT algorithms were developed later.

TOT algorithms became important with the development of GPS. GLONASS and Galileo employ similar concepts. The primary complicating factor for all GNSSs is that the stations (transmitters on satellites) move continuously relative to the Earth. Thus, in order to compute its own position, a user's navigation receiver must know the satellites' locations at the time the information is broadcast in the receiver's time scale (which is used to measure the TOAs). To accomplish this: (1) satellite trajectories and TOTs in the satellites' time scales are included in broadcast messages; and (2) user receivers find the difference between their TOT and the satellite broadcast TOT (termed the clock bias or offset). GPS satellite clocks are synchronized to UTC (to within a published offset of a few seconds), as well as with each other. This enables GPS receivers to provide UTC time in addition to their position.

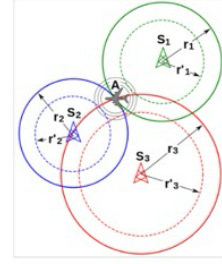


Fig. 2. Multilateration surveillance system TOT algorithm concept

Measurement geometry and related factors

Rectangular/Cartesian coordinates

Consider an emitter (E in Figure 3) at an unknown location vector

$$\vec{E} = (x, y, z),$$

which we wish to locate (surveillance problem). The source is within range of $m = n + 1$ receivers at known locations

$$\vec{P}_0, \vec{P}_1, \dots, \vec{P}_i, \dots, \vec{P}_n.$$

The subscript i refers to any one of the receivers:

$$\vec{P}_i = (x_i, y_i, z_i),$$

$$0 \leq i \leq n.$$

The distance (R_i) from the emitter to one of the receivers in terms of the coordinates is

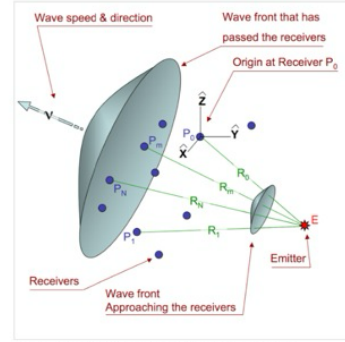


Fig. 3. Surveillance system TDOA geometry

$$R_i = |\vec{P}_i - \vec{E}| = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}. \quad (1)$$

For some solution algorithms, the math is made easier by placing the origin at one of the receivers (P_0), which makes its distance to the emitter

$$R_0 = \sqrt{(0 - x)^2 + (0 - y)^2 + (0 - z)^2} = \sqrt{x^2 + y^2 + z^2}. \quad (2)$$

Spherical coordinates

Low-frequency radio waves follow the curvature of the Earth (great-circle paths) rather than straight lines. In this situation, equation 1 is not valid. Loran-C^[12] and Omega^[13] are examples of systems that use spherical ranges. When a spherical model for the Earth is satisfactory, the simplest expression for the central angle (sometimes termed the *geocentric angle*) θ_{vi} between vehicle v and station i is

$$\cos \theta_{vi} = \sin \varphi_v \sin \varphi_i + \cos \varphi_v \cos \varphi_i \cos(\lambda_v - \lambda_i),$$

where latitudes are denoted by φ , and longitudes are denoted by λ . Alternative, better numerically behaved equivalent expressions can be found in great-circle navigation.

Analysis / validation of: 'Pseudo-range multilateration' wikipedia.com [2]

Virtually always, the coordinate frame is selected based on the wave trajectories. Thus, two- or three-dimensional Cartesian frames are selected most often, based on straight-line (line-of-sight) wave propagation. However, polar (also termed circular/spherical) frames are sometimes used, to agree with curved earth-surface wave propagation paths. Given the frame type, the origin and axes orientation can be selected, e.g., based on the station locations. Standard coordinate frame transformations may be used to place results in any desired frame. For example, GPS receivers generally compute their position using rectangular coordinates, then transform the result to latitude, longitude and altitude.

TDOA formation

Given m received signals, TDOA systems form $m - 1$ differences of TOA pairs (see "Calculating TDOAs or TOAs from received signals" below). All received signals must be a member of at least one TDOA pair, but otherwise the differences used are arbitrary (any two of the several sets of TDOAs can be related by an invertible linear transformation). Thus, when forming a TDOA, the order of the two TOAs involved is not important.

Some operational TDOA systems (e.g., Loran-C) designate one station as the "master" and form their TDOAs as the difference of the master's TOA and the $m - 1$ "secondary" stations' TOAs. When $m = 3$, there are 3 possible TDOA combinations, each corresponding to a station being the de facto master. When $m = 4$, there are 16 possible TDOA sets, 12 of which do not have a de facto master. When $m = 5$, there are 135 possible TDOA sets, 130 of which do not have a de facto master. ✓

TDOA principle / surveillance

If a pulse is emitted from a vehicle, it will generally arrive at slightly different times at spatially separated receiver sites, the different TOAs being due to the different distances of each receiver from the vehicle. However, for given locations of any two receivers, a set of emitter locations would give the same time difference (TDOA). Given two receiver locations and a known TDOA, the locus of possible emitter locations is one half of a two-sheeted hyperboloid.

In simple terms, with two receivers at known locations, an emitter can be located onto one hyperboloid (see Figure 1).^[10] Note that the receivers do not need to know the absolute time at which the pulse was transmitted – only the time difference is needed. However, to form a useful TDOA from two measured TOAs, the receiver clocks must be synchronized with each other.

Consider now a third receiver at a third location which also has a synchronized clock. This would provide a third independent TOA measurement and a second TDOA (there is a third TDOA, but this is dependent on the first two TDOAs and does not provide additional information). The emitter is located on the curve determined by the two intersecting hyperboloids. A fourth receiver is needed for another independent TOA and TDOA. This will give an additional hyperboloid, the intersection of the curve with this hyperboloid gives one or two solutions, the emitter is then located at one of the two solutions.

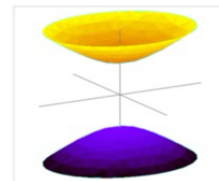


Fig 1. A two-sheeted hyperboloid

With four synchronized receivers there are 3 independent TDOAs, and three independent parameters are needed for a point in three dimensional space. (And for most constellations, three independent TDOAs will still give two points in 3D space). With additional receivers enhanced accuracy can be obtained. (Specifically, for GPS and other GNSSs, the atmosphere does influence the traveling time of the signal and more satellites does give a more accurate location). For an over-determined constellation (more than 4 satellites/TOAs) a least squares method can be used for 'reducing' the errors. Averaging over longer times can also improve accuracy.

The accuracy also improves if the receivers are placed in a configuration that minimizes the error of the estimate of the position.^[11]

The emitter may, or may not, cooperate in the multilateration surveillance process. Thus, multilateration surveillance is used with non-cooperating "users" for military and scientific purposes as well as with cooperating users (e.g., in civil transportation).

TDOA principle / navigation

Multilateration can also be used by a single receiver to locate itself, by measuring signals emitted from synchronized transmitters at known locations (stations). At least three emitters are needed for two-dimensional navigation (e.g., the Earth's surface); at least four emitters are needed for three-dimensional navigation. Although not true for real systems, for expository purposes, the emitters may be regarded as each broadcasting narrow pulses (ideally, impulses) at exactly the same time on separate frequencies (to avoid interference). In this situation, the receiver measures the TOAs of the pulses. In actual TDOA systems, the received signals are cross-correlated with an undelayed replica to extract the pseudo delay, then differenced with the same calculation for another station and multiplied by the speed of propagation to create range differences.

Several methods have been implemented to avoid self-interference. A historic example is the British Decca system, developed during World War II. Decca used the phase-difference of three transmitters. Later, Omega elaborated on this principle. For Loran-C, introduced in the late 1950s, all transmitters broadcast pulses on the same frequency with different, small time delays. GNSSs continuously transmitting on the same carrier frequency modulated by different pseudo random codes (GPS, Galileo, revised GLONASS).

TOT principle

Analysis / validation of: 'Pseudo-range multilateration' wikipedia.com [3]

The distance R_i from the vehicle to station i is along a great circle will then be

$$R_i = R_E \theta_{ei},$$

where R_E is the assumed radius of the Earth, and θ_{ei} is expressed in radians.

Time of transmission (user clock offset or bias)

Prior to GNSSs, there was little value to determining the TOT (as known to the receiver) or its equivalent in the navigation context, the offset between the receiver and transmitter clocks. Moreover, when those systems were developed, computing resources were quite limited. Consequently, in those systems (e.g., Loran-C, Omega, Decca), receivers treated the TOT as a nuisance parameter and eliminated it by forming TDOA differences (hence were termed TDOA or range-difference systems). This simplified solution algorithms. Even if the TOT (in receiver time) was needed (e.g., to calculate vehicle velocity), TOT could be found from one TOA, the location of the associated station, and the computed vehicle location.

With the advent of GPS and subsequently other satellite navigation systems: (1) TOT as known to the user receiver provides necessary and useful information; and (2) computing power had increased significantly. GPS satellite clocks are synchronized not only with each other but also with Coordinated Universal Time (UTC) (with a published offset) and their locations are known relative to UTC. Thus, algorithms used for satellite navigation solve for the receiver position and its clock offset (equivalent to TOT) simultaneously. The receiver clock is then adjusted so its TOT matches the satellite TOT (which is known by the GPS message). By finding the clock offset, GNSS receivers are a source of time as well as position information. Computing the TOT is a practical difference between GNSSs and earlier TDOA multilateration systems, but is not a fundamental difference. To first order, the user position estimation errors are identical.^[14]

TOA adjustments

Multilateration system governing equations – which are based on "distance" equals "propagation speed" times "time of flight" – assume that the energy wave propagation speed is constant and equal along all signal paths. This is equivalent to assuming that the propagation medium is homogeneous. However, that is not always sufficiently accurate; some paths may involve additional propagation delays due to inhomogeneities in the medium. Accordingly, to improve solution accuracy, some systems adjust measured TOAs to account for such propagation delays. Thus, space-based GNSS augmentation systems – e.g., Wide Area Augmentation System (WAAS) and European Geostationary Navigation Overlay Service (EGNOS) – provide TOA adjustments in real time to account for the ionosphere. Similarly, U.S. Government agencies used to provide adjustments to Loran-C measurements to account for soil conductivity variations.

Calculating TDOAs or TOAs from received signals

Assume a surveillance system calculates the time differences (τ_i for $i = 1, 2, \dots, m-1$) of wavefronts touching each receiver. The TDOA equation for receivers i and 0 is (where the wave propagation speed is c and the true vehicle-receiver ranges are R_0 and R_i)

$$\begin{aligned} c\tau_i &= cT_i - cT_0, \\ c\tau_i &= R_i - R_0. \end{aligned} \quad \checkmark \quad (3)$$

The quantity cT_i is often termed a pseudo-range. It differs from the true range between the vehicle and station i by an offset, or bias, which is the same for every signal. Differencing two pseudo-ranges yields the difference of the same two true-ranges.

Figure 4a (first two plots) show a simulation of a pulse waveform recorded by receivers P_0 and P_1 . The spacing between E , P_1 and P_0 is such that the pulse takes 5 time units longer to reach P_1 than P_0 . The units of time in Figure 4 are arbitrary. The following table gives approximate time scale units for recording different types of waves:

Type of wave	Material	Time units
Acoustic	Air	1 millisecond
Acoustic	Water	1/2 millisecond
Acoustic	Rock	1/10 millisecond
Electromagnetic	Vacuum, air	1 nanosecond

The red curve in Figure 4a (third plot) is the cross-correlation function ($P_1 \star P_0$). The cross-correlation function slides one curve in time across the other and returns a peak value when the curve shapes match. The peak at time = 5 is a measure of the time shift between the recorded waveforms, which is also the τ value needed for equation 3.

Figure 4b shows the same type of simulation for a wide-band waveform from the emitter. The time shift is 5 time units because the geometry and wave speed is the same as the Figure 4a example. Again, the peak in the cross-correlation occurs at $\tau_1 = 5$.

Figure 4c is an example of a continuous, narrow-band waveform from the emitter. The cross-correlation function shows an important factor when choosing the receiver geometry. There is a peak at time = 5 plus every increment of the waveform period. To get one solution for the measured time difference, the largest space between any two receivers must be closer than one wavelength of the emitter signal. Some systems, such as the LORAN C and Decca mentioned at earlier (recall the same math works for moving receiver and multiple known

Analysis / validation of: 'Pseudo-range multilateration' wikipedia.com [4]

For finding a user's location in a two dimensional (2-D) Cartesian geometry, one can adapt one of the many methods developed for 3-D geometry, most motivated by GPS—for example, Bancroft's^[30] or Krause's.^[19] Additionally, there are specialized TDOA algorithms for two-dimensions and stations at fixed locations — notable is Fang's method.^[15]

A comparison of 2-D Cartesian algorithms for airport surface surveillance has been performed.^[31] However, as in the 3-D situation, it is likely the most utilized algorithms are based on Gauss–Newton NLLS.^{[16][14]}

Examples of 2-D Cartesian multilateration systems are those used at major airports in many nations to surveil aircraft on the surface or at very low altitudes.

Two-dimensional spherical algorithms

Razin^[29] developed a closed-form algorithm for a spherical Earth. Williams and Last^[32] extended Razin's solution to an osculating sphere Earth model.

When necessitated by the combination of vehicle-station distance (e.g., hundreds of miles or more) and required solution accuracy, the ellipsoidal shape of the Earth must be considered. This has been accomplished using the Gauss–Newton NLLS^[33] method in conjunction with ellipsoid algorithms by Andoyer,^[34] Vincenty^[35] and Sodano.^[36]

Examples of 2-D 'spherical' multilateration navigation systems that accounted for the ellipsoidal shape of the Earth are the Loran-C and Omega radionavigation systems, both of which were operated by groups of nations. Their Russian counterparts, CHAYKA and Alpha (respectively), are understood to operate similarly.

Cartesian solution with limited computational resources

Consider a three-dimensional Cartesian scenario. Improving accuracy with a large number of receivers (say, $n + 1$, numbered $0, 1, 2, \dots, n$) can be a problem for devices with small embedded processors, because of the time required to solve several simultaneous, non-linear equations (1, 2, 3). The TDOA problem can be turned into a system of linear equations when there are three or more receivers, which can reduce the computation time. Starting with equation 3, solve for R_i , square both sides, collect terms and divide all terms by $c\tau_i = R_i - R_0$:

$$\begin{aligned} R_i^2 &= (c\tau_i + R_0)^2, \\ 0 &= (c\tau_i) + 2R_0 + \frac{R_0^2 - R_i^2}{c\tau_i}. \end{aligned} \quad \begin{aligned} &\checkmark \\ &\checkmark \\ &\checkmark \end{aligned} \quad \begin{aligned} &0 = 2R_0 \cdot c\tau_i + c\tau_i^2 + R_0^2 - R_i^2 \end{aligned} \quad (4)$$

Removing the $2R_0$ term will eliminate all the square root terms. That is done by subtracting the TDOA equation of receiver $i = 1$ from each of the others ($2 \leq m \leq n$)

$$0 = c\tau_i - c\tau_1 + \frac{R_0^2 - R_i^2}{c\tau_i} - \frac{R_0^2 - R_1^2}{c\tau_1}. \quad \checkmark \quad (5)$$

Focus for a moment on equation 1. Square R_0 , group similar terms and use equation 2 to replace some of the terms with R_0 .

$$R_0^2 - R_i^2 = -x_i^2 - y_i^2 - z_i^2 + x \cdot 2x_i + y \cdot 2y_i + z \cdot 2z_i. \quad (x^2 + y^2 + z^2) - [(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2] \quad (6)$$

Combine equations 5 and 6, and write as a set of linear equations (for $2 \leq i \leq n$) of the unknown emitter location x, y, z

$$\begin{aligned} 0 &= xA_i + yB_i + zC_i + D_i, & 0 &= c\tau_i - c\tau_1 + [(-x_i^2 - y_i^2 - z_i^2 + x \cdot 2x_i + y \cdot 2y_i + z \cdot 2z_i) / c\tau_i] \\ &\quad - [(-x_1^2 - y_1^2 - z_1^2 + x \cdot 2x_1 + y \cdot 2y_1 + z \cdot 2z_1) / c\tau_1] \\ A_i &= \frac{2x_i}{c\tau_i} - \frac{2x_1}{c\tau_1}, \\ B_i &= \frac{2y_i}{c\tau_i} - \frac{2y_1}{c\tau_1}, \\ C_i &= \frac{2z_i}{c\tau_i} - \frac{2z_1}{c\tau_1}, \\ D_i &= c\tau_i - c\tau_1 - \frac{x_i^2 + y_i^2 + z_i^2}{c\tau_i} + \frac{x_1^2 + y_1^2 + z_1^2}{c\tau_1}. \end{aligned} \quad \checkmark \quad (7)$$

Use equation 7 to generate the four constants A_i, B_i, C_i, D_i from measured distances and time for each receiver $2 \leq i \leq n$. This will be a set of $n - 1$ inhomogeneous linear equations.
 Note: assumed constant over measurement period. Will change due to orbital dynamics of the satellites.

There are many robust linear algebra methods that can solve for (x, y, z) , such as Gaussian elimination. Chapter 15 in Numerical Recipes^[37] describes several methods to solve linear equations and estimate the uncertainty of the resulting values.

Analysis / validation of: 'Pseudo-range multilateration' wikipedia.com [5]

Iterative algorithms

The defining characteristic and major disadvantage of iterative methods is that a 'reasonably accurate' initial estimate of the 'vehicle's' location is required. If the initial estimate is not sufficiently close to the solution, the method may not converge or may converge to an ambiguous or extraneous solution. However, iterative methods have several advantages:^[16]

- Can use redundant measurements ($m > d + 1$)
- Can utilize uninvertible measurement equations — Enables, e.g., use of complex problem geometries such as an ellipsoidal earth's surface.
- Can utilize measurements lacking an analytic expression (e.g., described by a numerical algorithm and/or involving measured data) — What is required is the capability to compute a candidate solution (e.g., user-station range) from hypothetical user position quantities (e.g., latitude and longitude)
- Amenable to automated processing (avoids the extraneous and ambiguous solutions which occur in direct algorithms)
- Can treat random measurement errors linearly, which when $m > d + 1$ allows averaging and thus minimizes their effect on position error.

Many real-time multilateration systems provide a rapid sequence of user's position solutions — e.g., GPS receivers typically provide solutions at 1 sec intervals. Almost always, such systems implement: (a) a transient 'acquisition' (surveillance) or 'cold start' (navigation) mode, whereby the user's location is found from the current measurements only; and (b) a steady-state 'track' (surveillance) or 'warm start' (navigation) mode, whereby the user's previously computed location is updated based current measurements (rendering moot the major disadvantage of iterative methods). Often the two modes employ different algorithms and/or have different measurement requirements, with (a) being more demanding. The iterative Gauss-Newton algorithm is often used for (b) and may be used for both modes.

When there are more TOA measurements than the $d + 1$ unknown quantities — e.g., 5 or more GPS satellite TOAs — the iterative Gauss-Newton algorithm for solving non-linear least squares (NLLS) problems is often preferred. Except for pathological station locations, an over-determined situation eliminates possible ambiguous and/or extraneous solutions that can occur when only the minimum number of TOA measurements are available. Another important advantage of the Gauss-Newton method over some closed-form algorithms is that it treats measurement errors linearly, which is often their nature, thereby reducing the effect measurement errors by averaging. The Gauss-Newton method may also be used with the minimum number of measurements.

While the Gauss-Newton NLLS iterative algorithm is widely used in operational systems (e.g., ASDE-X), the Nelder-Mead iterative method is also available. Example code for the latter, for both TOA and TDOA systems, are available.^[38]

Accuracy

Multilateration is often more accurate for locating an object than true-range multilateration or triangulation, as (a) it is inherently difficult and/or expensive to accurately measure the true range (distance) between a moving vehicle and a station, particularly over large distances, and (b) accurate angle measurements require large antennas which are costly and difficult to site.

Accuracy of a multilateration system is a function of several factors, including:

- The geometry of the receiver(s) and transmitter(s) for electronic, optical or other wave phenomenon.
- The synchronization accuracy of the transmitters (navigation) or the receivers (surveillance), including the thermal stability of the clocking oscillators.
- Propagation effects — e.g., diffraction or reflection changes from the assumed line of sight or curvilinear propagation path.
- The bandwidth of the emitted signals—e.g., the rise-time of the pulses employed with pulse coded signals.
- Inaccuracies in the locations of the transmitters or receivers when used as known locations.

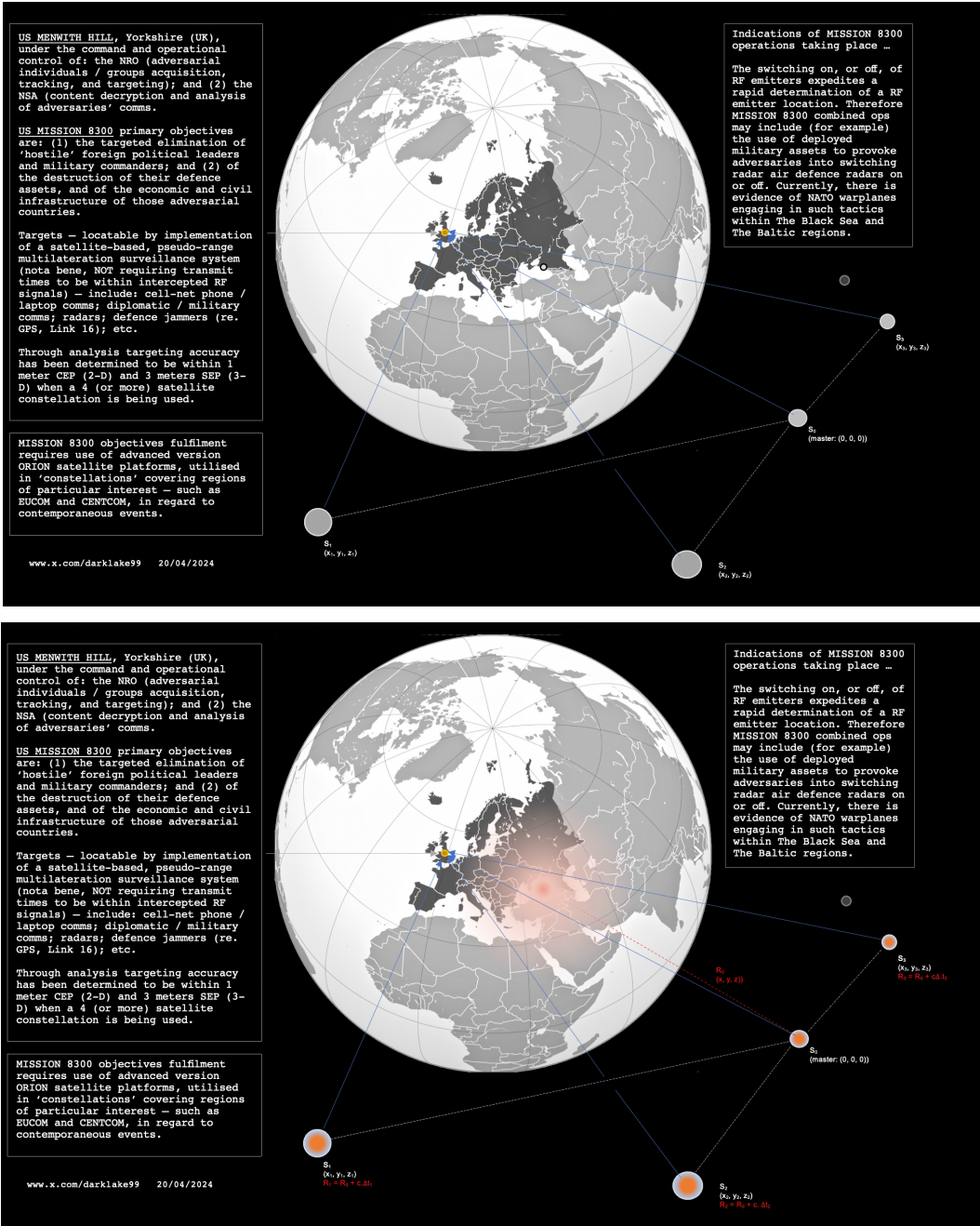
The accuracy can be calculated by using the Cramér-Rao bound and taking account of the above factors in its formulation. Additionally, a configuration of the sensors that minimizes a metric obtained from the Cramér-Rao bound can be chosen so as to optimize the actual position estimation of the target in a region of interest.^[11]

Concerning the first issue (user-station geometry), planning a multilateration system often involves a dilution of precision (DOP) analysis to inform decisions on the number and location of the stations and the system's service area (two dimensions) or volume (three dimensions). In a DOP analysis, the TOA measurement errors are assumed to be statistically independent and identically distributed. This reasonable assumption separates the effects of user-station geometry and TOA measurement errors on the error in the calculated user position.^{[2][39]}

Station synchronization

Multilateration requires that spatially separated stations — either transmitters (navigation) or receivers (surveillance) — have synchronized 'clocks'. There are two distinct synchronization requirements: (1) maintain synchronization accuracy continuously over the life expectancy of the system equipment involved (e.g., 25 years); and (2) for surveillance, accurately measure the time interval between TOAs for each 'vehicle' transmission. Requirement (1) is transparent to the user, but is an important system design consideration. To maintain synchronization, station clocks must be synchronized or reset regularly (e.g., every half-day for GPS, every few minutes for ASDE-X). Often the system accuracy is monitored continuously by "users" at known locations — e.g., GPS has five monitor sites.

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